Reply to Comment on "Thermal Model for Adaptive Competition in a Market": In our Letter [1] we introduced a generalization of the minority game (MG) [2], called the thermal minority game (TMG). One of the main new features was allowing for the stochastic decision making of the agents, controlled by a temperature T. In the completely deterministic case $T = 0$, the original MG is recovered.

In their Comment [3] Challet et al. claim that: (i) the equations of our model reduce to Eqs.(2) of [3]; (ii) Eqs.(2) of [3] lead to the "exact solution of the MG"; (iii) the crossover to a random value of the variance for $T \gg 1$ found in Figs. 2-3 of [1] is due to short waiting times in the simulations. Remarks (i) and (ii) are incorrect. Point (iii) is true, and highlights even more the crucial role of the temperature in the TMG. The central claim of [3] is that the effects of the temperature in the TMG "can be eliminated by time rescaling" and consequently the behaviour of the TMG is "*independent* of T". These statements have no general validity.

Challet et al. obtained their Eqs.(2) of [3] by averaging our equations both over the noise $\vec{\eta}$ and the individual strategy distribution. Averaging over $\vec{\eta}$ is legitimate, since it preserves the full macroscopic dynamics for all d and T. However, averaging over the individual strategies as in [3] is too naive and misses important correlated fluctuations. Besides, this procedure is conceptually wrong: replacing R_i^* by its average is equivalent to allowing the agents (who have s fixed strategies available) to play with *any* strategy formed by a linear combination of the s fixed ones. This is not sensible and contrary to the original spirit of the model [2,1].

In Fig. 1 we demonstrate the above assertions by comparing the results of simulations on the TMG with those resulting from the equations of Challet et al. Not surprisingly, for $T \sim 0$ the approximation leading to Eqs.(2) of [3] works well (at $T = 0$ there is no stochasticity and the average is equal to the best strategy). However, as soon as we turn on T, this approximation fails to reproduce the correct behaviour. Clearly, Eqs. (2) of [3] miss fluctuation effects and do not describe the behaviour of our system for all $T \neq 0$.

In [4] Challet et al. approximate the r.h.s. of Eqs.(2) of [3] as the gradient of an effective Hamiltonian $\mathcal{H}$ and study the MG by minimizing $\mathcal{H}$. This procedure is predicated on the assumption of equilibration of the strategy-use probabilities $\pi_i^a(t)$. This assumption is false for $d < d_c$ [5]. The consequence is that although their method correctly accounts for the phase $d > d_c$, it fails completely for $d < d_c$ (see Fig. 1 of [4]). Thus, to claim as Challet et al. do in (ii), that they have found the "exact solution of the MG" is incorrect and misleading.

Remark (iii) of the Comment is correct: the crossover to a random variance for large T observed in [1] is due to finite simulation times. The time required to reach the steady state for $T \gg 1$ is of order NT. This is a very interesting observation. It means that in the phase $d < d_c$, for any *finite* temperature larger than a critical value $T_c \sim 1$ [1], the performance of the system will be better than the original MG, provided that we wait long enough. In the inset of Fig.1 we show σ vs. T for waiting times much larger than NT for the values of d of Fig.2 of [1].

In other words, what Challet et al. have correctly pointed out in their Comment is that *any* degree of stochasticity above a given threshold makes the TMG perform better than the MG. The claim that "the collective behaviour should be *independent* of T" is clearly wrong. The remarkable structure of the TMG with the temperature begs for further investigation.

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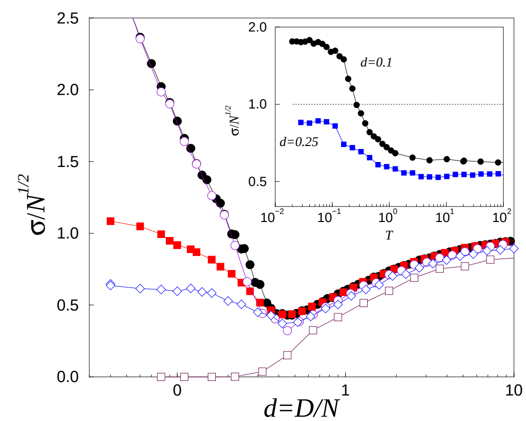

FIG. 1. σ vs. d in the TMG ($\bullet$ $T = 0$, $\blacksquare$ $T = 0.32$), and with the approximation of the Comment with σ calculated as in [4], $\sigma^2 = \sum_{ij} \langle \vec{R}_i^* \cdot \vec{R}_j^* \rangle$ ($\circ$ $T = 0.02$, $\diamond$ $T = 0.32$). Open squares are the quantity $\sum_{ij} \langle \vec{R}_i^* \rangle \cdot \langle \vec{R}_j^* \rangle$. $s = 2$, $N = 100$, $t = t_0 = 10^4$, 10^2 samples. INSET: σ vs. T for $d = 0.1$ and $d = 0.25$, for waiting time $t \sim 10^6 \gg NT$.